B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let f(x) = [x] (greatest integer function), and g(x) = |x|, where x in R (the set of real numbers). Evaluate (f o g)(1/3) – (g o f)(-1/3).

b) Find the generating function associated with the sequence 2, 6, 18, 54, …

c) How many 12-digit 0-1 strings contain precisely five 1's?

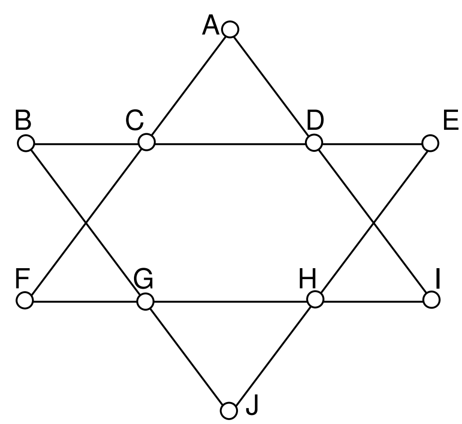
d) Find the O(f(n) where

i) $f(n)= 3\ n! – 17\ n^4$

ii) f(n)= 2 + 4 + 6 + … + 2n

e) Let Σ={a,b,c.} and let x=aabc. State whether or not x belongs to a*(b+c)*?

f) Is there an Eulerian circuit in the graph shown? If yes, find it. If not, explain why not?



g) Let A and B be nonempty sets. Prove that if A x B = B x A then A = B.

(7x4)

2.

a) Determine the validity of the following argument:
If I work hard, then I earn lots of money.
If I earn lots of money, then I pay high taxes.
Therefore, if I do not work hard, then I do not pay high taxes.

b) In a 12-day period, a small business mailed 195 bills to customers. Using the Pigeonhole principle show that during some period of three consecutive days at least 49 bills were mailed.

c) Solve the recurrence relation $a_n = 4\ a_{n-1} + 3n2^n$, n ≥ 1, given $a_0 = 4$.

(6+6+6)

3.

a) Let A= Q – {1} and '*' be an operation on A defined by *a* * b = a + b -ab for all a, b in A.

i) Find the identity element of A with respect to *.

ii) Find the inverse of elements of A with respect to *.

b) Consider all 3-digit number 000 to 999. In how many of these numbers are all the digits different?

c) Give a proof or provide a counter example that disproves the following statement:

"If n ≥ 1, $5^n + n + 1$ is divisible by 7".

(6+6+6)

4.

a) Suppose the first 4 digits of a telephone number of a particular zone of the city are fixed. The last 4 digits can be any number from {0, 1, 2, …, 9} and it must include at least one repeated digit. How many such telephone numbers are there?

b) Among the 30 students registered for a course in discrete mathematics, 15 students know the JAVA, 12 know C++, and 5 know both of these languages. Find

   i) How many students know at least one of JAVA or C++?

   ii) How many know only C++?

   iii) How many know exactly one of the languages JAVA and C++?

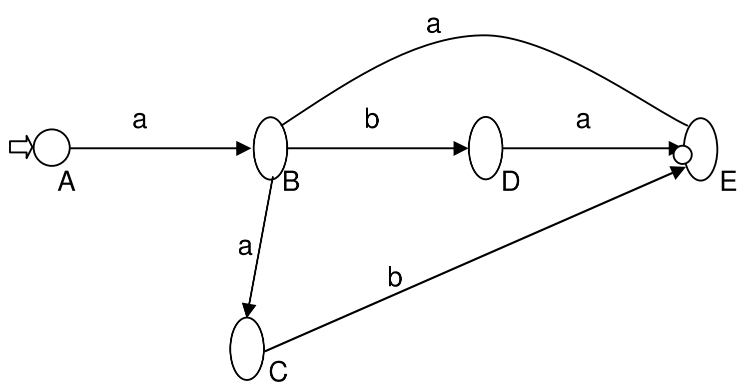
c) Prove or disprove the statement that $(p \wedge q) \wedge (p \rightarrow q)$ is a tautology.

(5+9+4)

5.

a) Sort the list 10, 11, 15, 3, 18, 14, 7, 1 into increasing order with a merge sort algorithm. Explain the step clearly.

b) Find the language accepted by the nondeterministic finite automata whose state diagram is given below:



c) Draw the Lattice diagram of the lattice of factors of 20 under divisibility.

(8+6+4)

6.

a) Show that K_4 and $K_{2,2}$ are planar graphs.

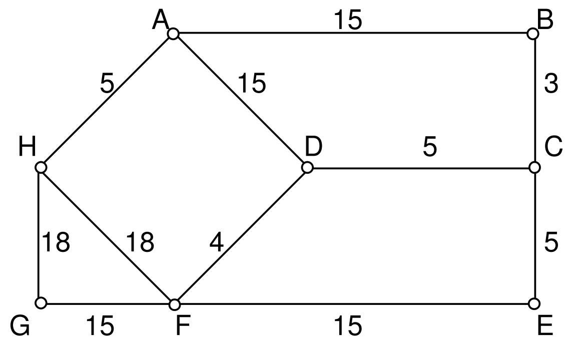
b) Using the Karnaugh map, simplify the following Boolean expression:

$$E(w, x, y, z) = wx'y'z + wxy'z' + wx'y'z' + w'x'yz + w'x'yz' + w'xyz + w'xyz'$$

(8+10)

7.

a) Determine the minimal cost railway network for the cities as shown in the graph:



b) Draw the state diagram for the Non-Deterministic Finite Automata (NDFA) for which the state table is given below. Find the languages accepted by this NDFA where S_1 and S_3 are accepting states.

I → / S ↓	f: a	f: b
S_0	S_2	S_1
S_1	S_1, S_2	S_3
S_2	Φ	Φ
S_3	S_2, S_3	S_2

(9+9)