

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

NOTE :

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1. (a) Evaluate $\lim_{x \rightarrow \infty} \sin^2 \frac{x}{2} \ln x$
 - (b) An urn contains 10 black and 10 white balls. Find the probability of drawing two balls of the same colour.
 - (c) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
 - (d) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$
 - (e) Does the infinite series $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges ?
 - (f) Find the foci, asymptotes and center in the equation $\frac{x^2}{4} - \frac{y^2}{5} = 1$.
 - (g) The probability density function $f(x)$ of a continuous random variable x is defined by $f(x) = \begin{cases} \frac{A}{x^3}, & 5 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$ find the value of A. [4x7]
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2. (a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ hence find A^{-1} .
 - (b) Find the area of the region in first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x -axis and the line $y = 2 - x$. [10+8]
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3. (a) Two independent samples of 8 and 7 items respectively had the following values of the variable (weight in ounces)
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|----------|----|----|----|----|----|---|----|----|
| Sample 1 | 9 | 11 | 13 | 11 | 15 | 9 | 12 | 14 |
| Sample 2 | 10 | 12 | 10 | 14 | 9 | 8 | 10 | - |
- Is the difference between the means of the sample significant?
- (b) The probability that man aged 60 will live to be 70 is 0.65 . What is the probability that out of 10 men, now 60, at least 7 will live to be 70 ? [9+9]

4. (a) Expand $\frac{x}{(e^x - 1)}$ in powers of x upto x^2 by Maclaurin's series.
- (b) Three urns contains 6 red, 4 black , 4 red, 6 black , 5 red, 5 black balls respectively. One of the urns is selected random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn.
- (c) Show that the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ represents a hyperbola. Find its centre, asymptotes and foci. [6+6+6]
5. (a) Find a value of c for $f(x) = x^2 - 4x - 3$ on the interval $[1, 4]$ using mean value theorem.
- (b) Solve the system of linear equations
 $x_1 - 2x_2 + 3x_3 = -2$
 $x_1 + x_2 - 2x_3 = 3$
 $2x_1 - x_2 + 3x_3 = 1$
- (c) Find the equation of the circle which passes through the points $(20, 3)$, $(19, 8)$ and $(2, -9)$. Find its centre and radius. [6+6+6]
6. (a) Find the horizontal asymptotes of the graph of $f(x) = \frac{x^3 - 2}{|x|^3 + 1}$.
- (b) Find a unit vector u in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.
- (c) Test the convergence of the infinite series $\sum_{n=2}^{\infty} \frac{1+n \ln n}{n^2 + 5}$. [6+6+6]
7. (a) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased.
- (b) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so where ? Also draw the graph. [9+9]

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