B0-R4 : BASIC MATHEMATICS

NOTE :

- 1. Answer question 1 and any FOUR questions from 2 to 7.
- 2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours

Total Marks : 100

1. (a) Express
$$\frac{(6+i)(2-i)}{(4+3i)(1-2i)}$$
 in the form of a+*i*b.

(b) Evaluate
$$\lim_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$$
.

(c) Prove that given matrix
$$A = \begin{bmatrix} 3 & 1+2i \\ 1-2i & 1 \end{bmatrix}$$
 is hermition matrix.

(d) Test the convergence of the following series

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots$$

(e) Solve the differential equation
$$\frac{dy}{x(2\log x + 1)} = \frac{dx}{\sin y + y\cos y}$$
.

(f) Find
$$\mathbf{u} \times \mathbf{v}$$
 and $\mathbf{v} \times \mathbf{u}$ if $\mathbf{u} = 2\vec{i} + \vec{j} + \vec{k}$ and $\mathbf{v} = -4\vec{i} + 3\vec{j} + \vec{k}$.

- (g) Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}, 1 \le x \le 4.$ (7x4)
- 2. (a) Solve the system of linear equations $x_1 + 2x_2 - x_3 = 1$, $3x_1 - 2x_2 + 2x_3 = 2$, $7x_1 - 2x_2 + 3x_3 = 5$ Using Gauss Eliminations Method.
 - (b) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line x = 3 about the line x = 3. (10+8)
- 3. (a) Find the area of the region in the plane enclosed by the cardioids $r = 2(1 + \cos\theta)$.
 - (b) Find the expansion of $\tan\left(x + \frac{\pi}{4}\right)$ in ascending powers of x up to terms in x^3 and find approximately the value of tan 43°. (9+9)

4. (a) Find the horizontal asymptotes of the graph of $f(x) = \frac{x^3 - 2}{|x|^3 + 1}$.

(b) Find the length of the circle of radius r defined the parametrically by $x = r \operatorname{cost}$ and $y = r \operatorname{sint}$, $0 \le t \le 2\pi$.

(c) Does the following series
$$\frac{3}{1^2-3} + \frac{3}{2^2-3} + \frac{3}{3^2-3} + \dots$$
 converges ? (6+6+6)

5. (a) Find a value of c for $f(x) = x^2 + 2x - 1$ on the interval [0, 1] using mean value theorem.

(b) Evaluate
$$\lim_{x \to 0} (2-x)^{\tan \frac{\pi x}{2}}$$
. (9+9)

- 6. (a) Using determinants, find the area of the triangle with vertices (-2, -3), (3, 2) and (-1, -8).
 - (b) Evaluate $\int_{1}^{e} \frac{\log x}{x} dx$.
 - (c) Find the center, radius, foci and asymptotes, vertices of the equation $x^2 + 4x + y^2 = 12$. (6+6+6)
- 7. (a) Solve the differential equation $ye^y dx = (y^3 + 2xe^y)dy$.
 - (b) Find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. (8+10)

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