No. of Printed Pages : **2**

Sl. No.

B3.2 - R4 : DISCRETE STRUCTURES

NOTE :

1. Answer question 1 and any FOUR from questions 2 to 7.

2. Parts of the same question should be answered together and in the same sequence.

Time : 3 Hours **Total Marks : 100**

1. (a) A travel agent surveyed 100 people to find out how many of them had visited the cities of Melbourne and Brisbane. Thirty-one people had visited Melbourne, 26 people had visited to Brisbane, and 12 people had visited both cities. Draw a Venn diagram to find the number of people who had visited :

(i) Melbourne or Brisbane

(ii) Brisbane but not Melbourne

(iii) Only one of the two cities

(iv) Neither city

(b) Prove that every cyclic group is abelian.

(c) Which of the following are true ? Prove your answer.

(i) $baa \in a^*b^*a^*b^*$

(ii) $a^*b^* \cap c^*d^* = \phi$

(iii) $abcd \in (a(cd)^*b)^*$

(d) Consider $x = 1101$ and $y = 0001$ to be elements of the Boolean algebra B4. Compute $y \wedge x$.

(e) Draw the Hasse diagram of the lattice $(D_{36}, |)$ consisting of all the divisors of 36 with partial order of divisibility.

(f) Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$. Find $n(A \cap B)$.

(g) Show that 101 is prime. **(7x4)**

2. (a) In the set of natural numbers N, define a relation R as follows : $\forall$ n, m $\in$ N, nRm if on division by 5 each of the integers n and m leaves the remainder less than 5, i.e. one of the numbers 0, 1, 2, 3 and 4. Show that R is equivalence relation. Also, obtain the pairwise disjoint subsets determined by R.

(b) Which of the following relations on {0, 1, 2, 3} are partial orderings ? Determine the properties of a partial ordering that the others lack.

(i) {(0, 0), (1, 1), (2, 2), (3, 3)}

(ii) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

(iii) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)}

(iv) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)} **(9+9)**

3. (a) Show that $(p \rightarrow q) \wedge (q \rightarrow p)$ is logically equivalent to $p \leftrightarrow q$.

(b) Use Karnaugh map to simplify the expression $xy + x'y'z' + x'yz'$.

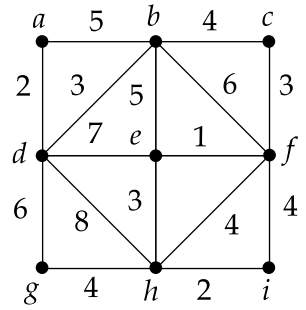
(c) Use mathematical induction to prove that $1+2+3+\ldots+N = \frac{N(N+1)}{2}$. **(6+6+6)**

4. (a) Let n and d be positive integers. How many positive integers not exceeding n are divisible by d ?

(b) A generation plant has two 20 MW generating units. The units always produce rated power unless they fail. The probability of failure of the 2 units are 0.08 and 0.05 respectively. What is the probability of getting 40 MW power from the plant ?

(c) A bag contains 10 red marbles, 10 white marble and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color ? **(6+6+6)**

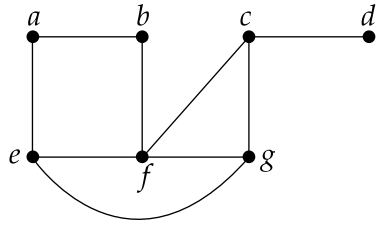
5. (a) Use Prim's algorithm to find minimum spanning tree for the given weighted graph.



(b) How many simple non-isomorphic graphs are possible with 3 vertices ? **(10+8)**

6. (a) Solve the recurrence relation $F_n = 5F_{n-1} - 6F_{n-2}$ where $F_0 = 1$ and $F_1 = 4$.

(b) Find a spanning tree of the simple graph G shown in following figure.



(9+9)

7. (a) Find a Turing machine that recognizes the set of bit strings that have a 1 as their second bit, that is, the regular set $(0 \cup 1)1(0 \cup 1)^*$.

(b) The number of edges in a planar bipartite graph of order n is at most 2n-4. **(10+8)**

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